Numerical Study of the Transient Heat Transfer within Thermal Energy Storage System based PCM

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ABSTRACT

In this paper, the unsteady state heat transfer through the thermal energy storage system is investigated numerically by using finite difference method in order to study the optimum thermal design of the latent heat thermal storage. The present analysis takes into account the heat transfer in two dimensions for fluid, tube and the PCM. The conical shape of the PCM is examined here for different diameters depending on the cone angle (ψ). The results of the present study are compared with other study to confirm the accuracy. Results of a numerical case study are presented and discussed. The results show that the decreasing the value of (ψ) will increase the hot and cold fluids effectivenesses and the difference between the effectiveness (hot and parallel cold and counter cold) increased with decreased value of (ψ) in the negative range (ψ = -0.13 deg). The counter flow for the cold fluid produce higher effectiveness than the parallel flow for the cold fluid especially for the negative value of (ψ = - 0.13 - 0 deg).

Keywords: Transient heat transfer, Thermal energy storage, Finite difference, PCM

الدراسة العددية للحالة غير المستقرة لنظام خزن الطاقة الحرارية

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الخلاصة

خلال هذه الدراسة، الحالة الغير مستقرة لانتقال الحرارة خلال نظامُ خزنِ الطاقة الحرارية تم دراستها بطريقة عدديه باستعمال طريقةِ الفروقات المحدودةِ (Finite Difference) لغرض دراسة التصميمَ الحراريَ الأمثل لمنظومة خزن الطاقة الحرارية. خلال الدراسة الحالية تم اعتماد انتقال الحرارة ببعدين للسائل، وللأنبوب و للـ(PCM). الشكل المخروطي للـ(PCM) اخذ بنظر الاعتبار لقيم مختلفة من الأقطار المختلفةِ اعتمادا على زاوية ميل المخروط (ψ). نَتائِجَ الدراسةِ الحاليةِ قورنت مع دراسةِ أخرى لغرض تأكيد الدقة للتحليل العددي. نَتائِج الدراسة تم تدوينها ومناقشتها. النَتائِجَ توضح بان انخفاض قيمة (ψ) سَتَزِيدُ من قيمة الفاعلية للمائع البارد والساخن، والفرق بين الفاعليتين (السائل الساخن والبارد) يزداد بانخفاض قيمة (ψ) وخاصة بالقيم السالبة (ψ = -0.13 deg). جريان المائع البارد بالاتجاه المعاكس للمائع الحار ينتج فاعلية أعلى من جريان المائع البارد بالاتجاه الموازي وبالأخص عندما تكون (ψ) ذو قيم سالبه (ψ = -0.13 – 0 deg) .

Nomenclature

C	Constant mass heat capacity (kJ/kg.K)	r	r- direction (m)
Fo	Fourier Number (Eq. 7)	R	Dimensionless r- direction
h	Heat transfer coefficient (W/m^2.K)	Re	Reynolds number (Eq. 7)
H	Specific enthalpy (kJ/kg.K)	r_i	Inner radius for the inner tube (m)
k	Thermal conductivity (W/m.K)	R_i	Inner radius for PCM (m)
L	Length of the heat exchanger (m)	r_o	Outer radius for the inner tube (m)
L_d	Dimensionless length (Eq. 7)	t	Time (sec)
N_r	Numbers of divisions for the fluid in r- direction	T	Temperature (˚C)
N_{rp}	Numbers of divisions for the PCM in r- direction	t_{con}	Constant time
N_{rt}	Numbers of divisions for the cylinder wall in r- direction	u	Velocity (m/sec)
Nu	Nusselt number (Eq. 7)	V	Volume (m^3)
N_z	Numbers of divisions in z- direction	z	z- direction (m)
Pe	Peclet number (Eq. 7)	Z	Dimensionless z- direction
Pr	Prandtle number (Eq. 7)	E_p	The cumulative stored energy (J)
Ex_m	The cumulative exergy stored in the PCM (J)	Ex_f	The Exergy Transferred by fluid (J)

Greek

μ	Dynamic viscosity (Pa.sec)	υ	Kinematics viscosity (m^2/sec)
ρ	Density (Kg/m^3)	ε	effectiveness
ζ	Subscript for fluid velocity (m/s)	α	Fluid thermal diffusion (m^2/s)
ϕ	Dimensionless temperature	δ	Dimensionless ($\Delta Y/\Delta Z$)
τ	Dimensionless time	θ	Temperature difference (˚C)
ψ	Angle (deg.)	Δ	Change
∞	Environment		

Subscript

f	Fluid	j	Refer to z - direction
fc	Cold fluid	L	Liquid in PCM
fci	Inlet cold fluid	m	Melting
fh	Hot fluid	p	PCM
fhi	Inlet hot fluid	s	Sold in PCM
i	Refer to r - direction	w	Cylinder wall

Superscript

'''	Per unit volume	n	Refer to time

Abbreviation

PCM	Phase Change Material	TES	Thermal Energy Storage
STES	Sensible Thermal Energy Storage	LTES	Latent Thermal Energy Storage
LHTS	Latent Heat Thermal Storage Systems	HTF	heat transfer fluid
SHTS	Sensible Heat Thermal Storage Systems		

1. INTRODUCTION

The quantity of the stored energy is a one of the main limitations of the thermal solar energy using, therefore using of an effective thermal energy storage system is a common solution. Thermal energy storage (TES) refers to system that store energy in a thermal reservoir for later reuse. Many types of TES for heating, cooling and other applications exist (Jose et al, [2007]). There are two main types of TES: sensible (STES) (the temperature of the storage medium varies when heat is added or removed) and latent (LTES) (the temperature remains constant during the phase change of the storage medium). LTES has been proven to be a more effective method of storing energy (Keshavarz and et al, [2003]).

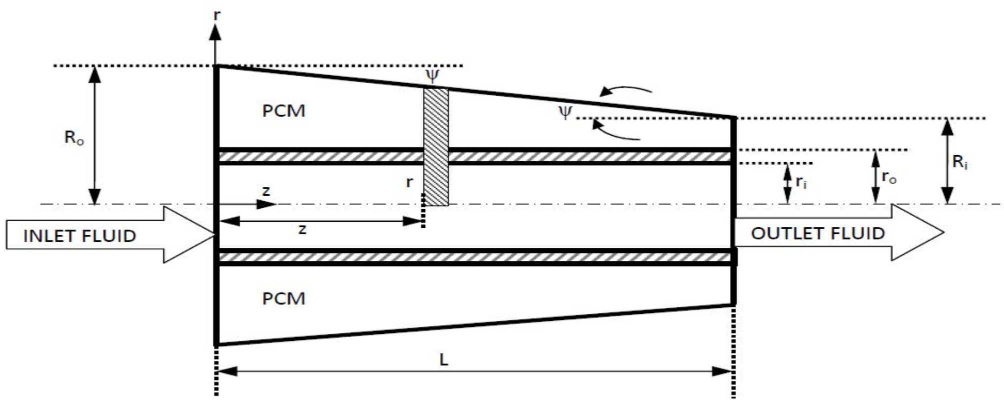
The LTES was studied in many researches. Keshavarz et al, [2003], investigated numerically the irreversibility for the shell and tube LHTS. Their results show that the irreversibility of thermal storage module is strongly affected by the size of PCM (diameter and length of the external cylinder) and melting temperature. Verma et al, [2008], analyzed thermodynamically LHTS to estimate the thermodynamic efficiency based on the first law of thermodynamics. MacPhee and Dincer, [2009], analyzed thermally (heat transfer and thermodynamic) the process of charging of an encapsulated ice thermal energy storage device. Their results showed the thermal exergy efficiencies varied between 40% and 93% for variable charging times.

The problem of low thermal conductivity of the phase change material (PCM) is analyzed with different techniques. The first technique proposed by adding fins; Shatikian et al, [2005], studied numerically the melting process of PCM in a heat storage unit with internal fins and the PCM is stored between the fins. Shatikian et al, [2008], studied numerically the transient melting of PCM in a heat sink with vertical internal fins and PCM is stored between the fins. They showed that the transient phase change process depends on the heat flux from the base, heat capacity of the PCM, and fin dimensions.

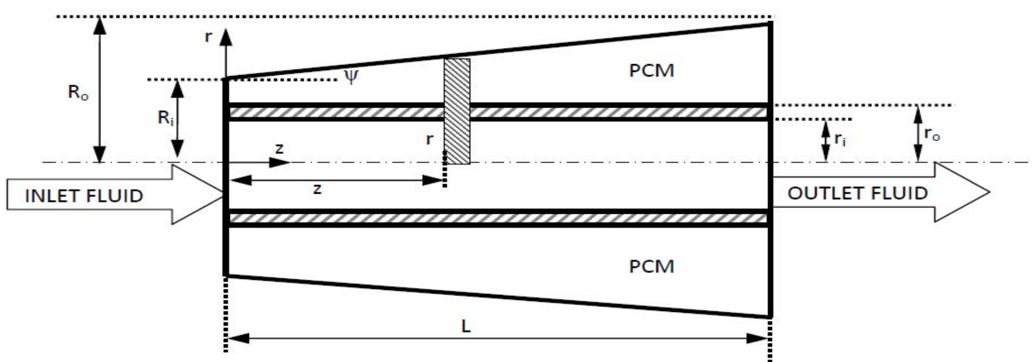
The second technique proposed was by using multiple PCMs; Aghbalou et al, [2006], studied analytically the exergetic optimization of a solar thermal energy system, that consists of a solar collector and a rectangular water storage tank that contains a multiple PCMs distributed in an assembly of slabs.

The third techniques proposed was with the help of additives for increasing the thermal conductivity of conventional PCMs; Mettawee and Assassa, [2007], investigated experimentally the influence of aluminum particles on melting and solidification processes of paraffin in solar collector. They showed that the time required for charging and discharging processes could be reduced substantially by adding the aluminum particles. Seeniraj et al, [2008], investigated the effect of high conductivity particles dispersed in the PCM on the performance during the melting process. They showed there exists an optimum fraction for the particles beyond which, the energy storage capacity would reduce. Zhong et al, [2010], analyzed the addition of the compressed expanded natural graphite matrices with different densities to increase the thermal conductivity of paraffin wax. Their results indicate that the thermal conductivity of the composites can be 28–180 greater than that of the pure paraffin wax. Jegadheeswaran and Pohekar, [2010], studied mathematically the effect of the dispersion of high conductivity particles on the melting rate and energy /exergy storage in a LHTS unit. They presented the comparison between the performance of particle dispersed unit and that of conventional pure PCM charged unit for different mass flow rates and inlet temperatures of HTF. Their results showed a significant improvement in the performance of the LHTS unit when high conductivity particles are dispersed. Recently, Gopal et al, [2010], described the energy and exergy analysis of a diesel engine integrated with a PCM based energy storage system. They found the energy efficiency of the integrated system is varying between 3.19% and 34.15%.

The purpose of the present study is to calculate the heat transfer through TES system numerically by using finite difference method. The TES system that was used in the present study consists of cylinder surrounded by a hollow cone. The water flows inside the inner pipe and the system works periodically. The annular space is filled with phase change material (PCM) (table 1). The cone shape of the PCM are examined for different diameters depending on the cone angle (ψ) .



For positive value of (ψ)



For nagative value of (ψ)
Figure (1), The TES case study

2. MATHEMATICAL MODEL

2.1 Assumptions

The following assumptions are used in the development of the numerical model:

1. The following fluids (hot and cold) are assumed fluid with constant and uniform hydrodynamic properties.
2. Two cases are taken in account for the cold water path; parallel flow (the cold fluid flow after the hot fluid in the direction of the hot fluid) and counter flow (the cold fluid flow after the hot fluid in the direction opposite to the direction of the hot fluid).
3. The fluid that flows in inner tube is incompressible.
4. External walls of TES are insulated.

5. Unsteady state condition and the system are symmetrical in the radial direction.

2.2 Governing Equations

A short presentation of the governing equations for the axisymmetric present system is as follows:

2.2.1 The energy equation for the fluids

The main governing equation for the fluid is derived from Navier- Stoke energy equation in two dimensional cylindrical coordinates (Figure (1)), [Deyi Shang, 2006], and in the dimensionless form, the equation can be written as (Appendix - A):

$$\frac{\partial^2\phi_f}{\partial Z^2}+\frac{\partial^2\phi_f}{\partial R^2}+\frac{1}{R}\frac{\partial\phi_f}{\partial R}=\frac{1}{Fo_f}\frac{\partial\phi_f}{\partial\tau}+Re_f\,Pr_f\,\frac{\partial\phi_f}{\partial Z} \qquad ...(1)$$

2.2.2 The energy equation for the cylinder wall

In dimensionless form, the energy equation for the cylinder wall can be written as following (Appendix- A):

$$\frac{\partial^2\phi_w}{\partial Z^2}+\frac{\partial^2\phi_w}{\partial R^2}+\frac{1}{R}\frac{\partial\phi_w}{\partial R}=\frac{1}{Fo_w}\frac{\partial\phi_w}{\partial\tau} \qquad \ldots(2)$$

2.2. 3 The energy equation for PCM

In dimensionless form, the energy equation for PCM can be written as (Appendix- A):

$$\frac{\partial^2\phi_p}{\partial Z^2}-\frac{\tan(\psi)}{R}\frac{\partial\phi_p}{\partial Z}+\frac{\partial^2\phi_p}{\partial R^2}+\frac{1}{R}\frac{\partial\phi_p}{\partial R}=\frac{1}{Fo_p}\frac{\partial\phi_p}{\partial\tau} \qquad ...(3)$$

Equation (3) is valid only when the temperature of PCM (ϕ_p) is not equal to the PCM melting temperature (ϕ_m).

The dimensionless quantities in the previous equations, as following:

$$\left\{\begin{array}{l} Z=\frac{z}{r_i},\quad R=\frac{r}{r_i},\quad L_d=\frac{L}{r_i},\quad Fo_f=\frac{t_{con}\alpha_f}{r_i^2},\quad \tau=\frac{t}{t_{con}},\quad Pe_f=Re_f\,Pr_f,\quad Re_f=\frac{u_f r_i}{\nu_f},\\ Pr_f=\frac{\nu_f}{\alpha_f},\quad Fo_w=\frac{t_{con}\alpha_w}{r_i^2},\quad Fo_p=\frac{t_{con}\alpha_p}{r_i^2},\quad Nu_f=\frac{h_f r_i}{k_w},\quad \phi_f(z,r,t)=\frac{T_f(z,r,t)-T_{fci}}{T_{fhi}-T_{fci}}=\frac{\theta_f(z,r,t)}{\theta_{fhc}},\\ \phi_w(z,r,t)=\frac{T_w(z,r,t)-T_{fci}}{T_{fhi}-T_{fci}}=\frac{\theta_w(z,r,t)}{\theta_{fhc}},\quad \phi_p(z,r,t)=\frac{T_p(z,r,t)-T_{fci}}{T_{fhi}-T_{fci}}=\frac{\theta_p(z,r,t)}{\theta_{fhc}} \end{array}\right\} \qquad \ldots(4)$$

2.3 Initial and Boundary Conditions

For the fluid

$$\phi_f(R,Z,0)=\left\{\begin{array}{lll}0 & \text{Discharged Process} & \text{(Cold fluid)}\\ 1 & \text{Charged Process} & \text{(Hot fluid)}\end{array}\right\}\quad 0\le R\le 1,\ 0\le Z\le L_d,\ \tau=0 \qquad ...(5)$$

$$\phi_f(R,0,\tau)=\begin{cases}0 & \text{Discharged Process} \quad (\text{Cold fluid})\\ 1 & \text{Charged Process} \quad (\text{Hot fluid})\end{cases}\quad 0\le R\le 1,\ Z=0,\ 0<\tau\le\tau_{max} \quad \text{... (6)}$$

$$\frac{\partial\phi_f(0,Z,\tau)}{\partial R}=0 \qquad R=0,\quad 0\le Z\le L_d,\ 0<\tau\le\tau_{max} \quad \text{... (7)}$$

$$\frac{\partial\phi_f(1,Z,\tau)}{\partial R}=\frac{k_w}{k_f}\frac{\partial\phi_w(1,Z,\tau)}{\partial R}\ \ \&\ \phi_f(1,Z,\tau)=\phi_w(1,Z,\tau) \qquad R=1,\ 0\le Z\le L_d,\ 0<\tau\le\tau_{max} \quad \text{... (8)}$$

For the wall

$$\phi_w(R,Z,0)=\begin{cases}0 & \text{Discharged Process} \ (\text{Cold fluid})\\ 1 & \text{Charged Process} \quad (\text{Hot fluid})\end{cases}\quad 1\le R\le\frac{r_o}{r_i},\quad 0\le Z\le L_d,\ \tau=0 \quad \text{... (9)}$$

$$\frac{\partial\phi_w(R,0,\tau)}{\partial Z}=0 \qquad 1\le R\le\frac{r_o}{r_i},\quad Z=0,\ 0<\tau\le\tau_{max} \quad \text{... (10)}$$

$$\frac{\partial\phi_w(R,L_d,\tau)}{\partial Z}=0 \qquad 1\le R\le\frac{r_o}{r_i},\quad Z=L_d,\ 0<\tau\le\tau_{max} \quad \text{... (11)}$$

$$\frac{\partial\phi_w(1,Z,\tau)}{\partial R}=\frac{k_f}{k_w}\frac{\partial\phi_f(1,Z,\tau)}{\partial R}\ \ \&\ \ \phi_f(1,Z,\tau)=\phi_w(1,Z,\tau) \qquad R=1,\ 0\le Z\le L_d,\ 0<\tau\le\tau_{max} \quad \text{... (12)}$$

$$\frac{\partial\phi_w(\frac{r_o}{r_i},Z,\tau)}{\partial R}=\frac{k_p}{k_w}\frac{\partial\phi_p(\frac{r_o}{r_i},Z,\tau)}{\partial R}\ \&\ \phi_w(\frac{r_o}{r_i},Z,\tau)=\phi_p(\frac{r_o}{r_i},Z,\tau) \qquad R=\frac{r_o}{r_i},\ 0\le Z\le L_d,\ 0<\tau\le\tau_{max} \quad \text{... (13)}$$

For the PMC

$$\phi_p(R,Z,0)=\begin{cases}0 & \text{Discharged Process} \ (\text{Cold fluid})\\ 1 & \text{Charged Process} \quad (\text{Hot fluid})\end{cases}\quad \frac{r_o}{r_i}\le R\le\frac{r}{r_i},\quad 0\le Z\le L_d,\ \tau=0 \quad \text{... (14)}$$

$$\frac{\partial\phi_p(R,0,\tau)}{\partial Z}=0 \qquad \frac{r_o}{r_i}\le R\le\frac{r}{r_i},\quad Z=0,\ 0<\tau\le\tau_{max} \quad \text{... (15)}$$

$$\frac{\partial\phi_p(R,L_d,\tau)}{\partial Z}=0 \qquad \frac{r_o}{r_i}\le R\le\frac{r}{r_i},\quad Z=L_d,\ 0<\tau\le\tau_{max} \quad \text{... (16)}$$

$$\frac{\partial\phi_p(r/r_i,Z,\tau)}{\partial R}=0 \qquad R=\frac{r}{r_i},\quad Z=L_d,\ 0<\tau\le\tau_{max} \quad \text{... (17)}$$

$$\frac{\partial\phi_p(\frac{r_o}{r_i},Z,\tau)}{\partial R}=\frac{k_w}{k_p}\frac{\partial\phi_w(\frac{r_o}{r_i},Z,\tau)}{\partial R}\ \&\ \phi_w(\frac{r_o}{r_i},Z,\tau)=\phi_p(\frac{r_o}{r_i},Z,\tau) \qquad R=\frac{r_o}{r_i},\ 0\le Z\le L_d,\ 0<\tau\le\tau_{max} \quad \text{... (18)}$$

3. NUMERICAL ANALYSIS

Finite difference method has used as numerical method to solve the governing equations.

3.1 Finite Difference

The finite difference technique (explicit method) is used to solve the governing equations with initial and boundary conditions, as following (Appendix – B):

For the fluid

$$\phi^{n+1}_{f(i,j)} = \begin{Bmatrix} (1 - \frac{Fo_f \Delta\tau Pe_f}{\Delta Z} - \frac{Fo_f \Delta\tau}{(i-1)\Delta R^2} - 2\frac{Fo_f \Delta\tau}{\Delta Z^2} - 2\frac{Fo_f \Delta\tau}{\Delta R^2})\phi^n_{f(i,j)} + (\frac{Fo_f \Delta\tau Pe_f}{\Delta Z} + \frac{Fo_f \Delta\tau}{\Delta Z^2})\phi^n_{f(i,j-1)} \\ + \frac{Fo_f \Delta\tau}{\Delta R^2}\frac{i}{(i-1)}\phi^n_{f(i+1,j)} + \frac{Fo_f \Delta\tau}{\Delta Z^2}\phi^n_{f(i,j+1)} + \frac{Fo_f \Delta\tau}{\Delta R^2}\phi^n_{f(i-1,j)} \end{Bmatrix} \quad ...(19)$$

For the cylinder wall

$$\phi^{n+1}_{w(i,j)} = (1 - 2\frac{\Delta\tau Fo_w}{\Delta Z^2} - 2\frac{\Delta\tau Fo_w}{\Delta R^2_w} - \frac{\Delta\tau Fo_w}{(i-1)\Delta R^2_w})\phi^n_{w(i,j)} + \frac{\Delta\tau Fo_w}{\Delta Z^2}(\phi^n_{w(i,j+1)} + \phi^n_{w(i,j-1)}) + \frac{\Delta\tau Fo_w}{\Delta R^2_w}[\frac{i}{(i-1)}\phi^n_{w(i+1,j)} + \phi^n_{w(i-1,j)}] \quad ...(20)$$

For the PCM

$$\phi^{n+1}_{p(i,j)} = \begin{Bmatrix} (1 - 2\frac{\Delta\tau Fo_p}{\Delta Z^2} - 2\frac{\Delta\tau Fo_p}{\Delta R^2_p} - \frac{\Delta\tau Fo_p \tan(\psi)}{(i-1)\Delta R_p \Delta Z} - \frac{\Delta\tau Fo_p}{(i-1)\Delta R^2_p})\phi^n_{p(i,j)} + \frac{\Delta\tau Fo_p}{\Delta Z^2}\phi^n_{p(i,j+1)} \\ + (\frac{\Delta\tau Fo_p \tan(\psi)}{(i-1)\Delta R_p \Delta Z} + \frac{\Delta\tau Fo_p}{\Delta Z^2})\phi^n_{p(i,j-1)} + \frac{\Delta\tau Fo_p}{\Delta R^2_p}\frac{i}{(i-1)}\phi^n_{p(i+1,j)} + \frac{\Delta\tau Fo_p}{\Delta R^2_p}\phi^n_{p(i-1,j)} \end{Bmatrix} \quad ...(21)$$

For the time interval ($\Delta\tau$)

The time interval (change) calculated from the equations (19, 20 and 21) as following:

$$\Delta\tau \le \begin{Bmatrix} \frac{1}{Fo_f(\frac{Pe_f}{\Delta Z} + \frac{1}{(i-1)\Delta R^2} + \frac{2}{\Delta Z^2} + \frac{2}{\Delta R^2})}, \quad \text{or} \quad \frac{1}{Fo_w(\frac{2}{\Delta Z^2} + \frac{2}{\Delta R^2_w} + \frac{1}{(i-1)\Delta R^2_w})}, \\ \text{or} \quad \frac{1}{Fo_p(\frac{2}{\Delta Z^2} + \frac{2}{\Delta R^2_p} + \frac{\tan(\psi)}{(i-1)\Delta R_p \Delta Z} + \frac{1}{(i-1)\Delta R^2_p})} \end{Bmatrix} \quad ...(22)$$

Smaller value for the ($\Delta\tau$) is selected to generate the solution convergence.

Boundary Conditions:

Fluid flow

$$\phi^n_{f(i,j)} = \begin{Bmatrix} 0 & \text{Uncharged Process (Cold fluid)} \\ 1 & \text{Charged Process (Hot fluid)} \end{Bmatrix} \quad \text{where } n = 0, \quad i = 1, N_r, \quad j = 1, N_z \quad \ldots (23)$$

$$\phi^n_{f(i,1)} = \begin{Bmatrix} 0 & \text{Uncharged Process (Cold fluid)} \\ 1 & \text{Charged Process (Hot fluid)} \end{Bmatrix} \quad \text{where } n = 0, \tau_{max}, \quad i = 1, N_r, \quad j = 1 \quad \ldots (24)$$

$$\frac{\partial \phi^n_{f(i,j)}}{\partial R} = 0 \qquad n = 0, \tau_{max}, \quad i = 1, \quad j = 2, N_z \quad \ldots (25)$$

$$\frac{\phi^n_{f(1,j+1)} - 2\phi^n_{f(1,j)} + \phi^n_{f(1,j-1)}}{\Delta Z^2} + \frac{2\phi^n_{f(2,j)} - 2\phi^n_{f(1,j)}}{\Delta R^2} = \frac{1}{Fo_f}\frac{\phi^{n+1}_{f(1,j)} - \phi^n_{f(1,j)}}{\Delta\tau} + Pe_f\frac{\phi^n_{f(1,j)} - \phi^n_{f(1,j-1)}}{\Delta Z}$$

$$\phi^{n+1}_{f(1,j)} = (1 - \frac{Fo_f \Delta\tau Pe_f}{\Delta Z} - 2\frac{Fo_f \Delta\tau}{\Delta Z^2} - 2\frac{Fo_f \Delta\tau}{\Delta R^2})\phi^n_{f(1,j)} + (\frac{Fo_f \Delta\tau Pe_f}{\Delta Z} + \frac{Fo_f \Delta\tau}{\Delta Z^2})\phi^n_{f(1,j-1)} + 2\frac{Fo_f \Delta\tau}{\Delta R^2}\phi^n_{f(2,j)} + \frac{Fo_f \Delta\tau}{\Delta Z^2}\phi^n_{f(1,j+1)} \quad ...(26)$$

$$\frac{\partial \phi^n_{f(N_r,j)}}{\partial R} = \frac{k_w}{k_f}\frac{\partial \phi^n_{w(1,j)}}{\partial R} \qquad n = 0, \tau_{max}, \quad i = N_r, \quad j = 2, N_z \quad ...(27)$$

$$\frac{\partial^2 \phi_f}{\partial Z^2} + \frac{1}{R}\frac{\partial}{\partial R}(R\frac{\partial \phi_f}{\partial R}) = \frac{1}{Fo_f}\frac{\partial \phi_f}{\partial \tau} + Re_f\, Pr_f\frac{\partial \phi_f}{\partial Z} \Rightarrow \frac{\partial^2 \phi_f}{\partial Z^2} + \frac{1}{R}\frac{k_w}{k_f}\frac{\partial \phi_w}{\partial R} + \frac{k_w}{k_f}\frac{\partial^2 \phi_w}{\partial R^2} = \frac{1}{Fo_f}\frac{\partial \phi_f}{\partial \tau} + Re_f\, Pr_f\frac{\partial \phi_f}{\partial Z}$$

$$\phi^{n+1}_{f(N_r,j)} = \begin{Bmatrix} (1 - 2\frac{Fo_f \Delta\tau}{\Delta Z^2} - \frac{Fo_f \Delta\tau Pe_f}{\Delta Z})\phi^n_{f(N_r,j)} + \frac{Fo_f \Delta\tau}{\Delta Z^2}\phi^n_{f(N_r,j+1)} + \\ \frac{Fo_f \Delta\tau}{\Delta R_w^2}\frac{(2N_r - 1)}{(N_r - 1)}\frac{k_w}{k_f}(\phi^n_{w(N_r+1,j)} - \phi^n_{w(N_r,j)}) + (\frac{Fo_f \Delta\tau}{\Delta Z^2} + \frac{Fo_f \Delta\tau Pe_f}{\Delta Z})\phi^n_{f(N_r,j-1)} \end{Bmatrix} \quad ...(28)$$

$$\phi^n_{f(N_r,j)} = \phi^n_{w(N_r,j)} \qquad n = 0, \tau_{max}, \quad i = N_r, \quad j = 2, N_z \quad ...(29)$$

Then, the equation (28) can be written:

$$\phi^{n+1}_{f(N_r,j)} = \begin{Bmatrix} (1 - 4\frac{Fo_f \Delta\tau}{\Delta Z^2} - \frac{Fo_f \Delta\tau Pe_f}{\Delta Z} - \frac{Fo_f \Delta\tau}{\Delta R_w^2}\frac{(2N_r - 1)}{(N_r - 1)}\frac{k_w}{k_f})\phi^n_{f(N_r,j)} + \frac{Fo_f \Delta\tau}{\Delta Z^2}\phi^n_{f(N_r,j+1)} \\ + \frac{Fo_f \Delta\tau}{\Delta R_w^2}\frac{(2N_r - 1)}{(N_r - 1)}\frac{k_w}{k_f}\phi^n_{w(2,j)} + (\frac{Fo_f \Delta\tau}{\Delta Z^2} + \frac{Fo_f \Delta\tau Pe_f}{\Delta Z})\phi^n_{f(N_r,j-1)} \end{Bmatrix} \quad ...(30)$$

For the wall

$$\phi^n_{w(i,j)} = \begin{Bmatrix} 0 & \text{Uncharged Process (Cold fluid)} \\ 1 & \text{Charged Process (Hot fluid)} \end{Bmatrix} \quad \text{where } n = 0, \quad i = 1, N_{rt}, \quad j = 1, N_z \quad ...(31)$$

$$\frac{\partial \phi^n_{w(i,1)}}{\partial Z} = 0 \qquad n = 0, \tau_{max}, \quad i = 2, N_{rt} - 1, \quad j = 1 \quad ...(32)$$

$$\phi^{n+1}_{w(i,1)} = (1 - 2\frac{\Delta\tau Fo_w}{\Delta Z^2} - 2\frac{\Delta\tau Fo_w}{\Delta R_w^2} - \frac{\Delta\tau Fo_w}{(i-1)\Delta R_w^2})\phi^n_{w(i,1)} + 2\frac{\Delta\tau Fo_w}{\Delta Z^2}\phi^n_{w(i,2)} + \frac{\Delta\tau Fo_w}{\Delta R_w^2}(\frac{i}{(i-1)}\phi^n_{w(i+1,1)} + \phi^n_{w(i-1,1)}) \quad ...(33)$$

$$\frac{\partial \phi^n_{w(i,N_z)}}{\partial Z} = 0 \qquad n = 0, \tau_{max}, \quad i = 2, N_{rt} - 1, \quad j = N_z \quad ...(34)$$

$$\phi^{n+1}_{w(i,N_z)} = (1 - 2\frac{\Delta\tau Fo_w}{\Delta Z^2} - 2\frac{\Delta\tau Fo_w}{\Delta R_w^2} - \frac{\Delta\tau Fo_w}{(i-1)\Delta R_w^2})\phi^n_{w(i,N_z)} + 2\frac{\Delta\tau Fo_w}{\Delta Z^2}\phi^n_{w(i,N_z-1)} + \frac{\Delta\tau Fo_w}{\Delta R_w^2}(\frac{i}{(i-1)}\phi^n_{w(i+1,N_z)} + \phi^n_{w(i-1,N_z)}) \quad ...(35)$$

$$\frac{\partial \phi^n_{w(1,j)}}{\partial R} = \frac{k_f}{k_w}\frac{\partial \phi^n_{f(N_r,j)}}{\partial R} \qquad n = 0, \tau_{max}, \quad i = 1 \quad j = 1, N_z \quad ...(36)$$

$$\frac{\partial^2 \phi_w}{\partial Z^2} + \frac{1}{R}\frac{\partial}{\partial R}(R\frac{k_f}{k_w}\frac{\partial \phi_f}{\partial R}) = \frac{1}{Fo_w}\frac{\partial \phi_w}{\partial \tau} \qquad \Rightarrow \qquad \frac{\partial^2 \phi_w}{\partial Z^2} + \frac{1}{R}\frac{k_f}{k_w}\frac{\partial \phi_f}{\partial R} + \frac{k_f}{k_w}\frac{\partial^2 \phi_f}{\partial R^2} = \frac{1}{Fo_w}\frac{\partial \phi_w}{\partial \tau}$$

$$\phi^{n+1}_{w(1,j)} = (1 - 2\frac{Fo_w \Delta\tau}{\Delta Z^2})\phi^n_{w(1,j)} + \frac{Fo_w \Delta\tau}{\Delta Z^2}(\phi^n_{w(1,j+1)} + \phi^n_{w(1,j-1)}) + \frac{Fo_w \Delta\tau}{\Delta R^2}\frac{k_f}{k_w}(\phi^n_{f(N_r+1,j)} + \frac{N_r}{(N_r - 1)}\phi^n_{f(N_r-1,j)} - \frac{(2N_r - 3)}{(N_r - 1)}\phi^n_{f(N_r,j)}) \quad ...(37)$$

$$\phi^n_{f(N_r,j)} = \phi^n_{w(1,j)} \qquad n = 0, \tau_{max}, \quad i = 1 \quad j = 1, N_z \quad ...(38)$$

Then, the equation (37), can be written:

$$\phi^{n+1}_{w(1,j)} = (1-2\frac{Fo_w\Delta\tau}{\Delta Z^2} - \frac{Fo_w\Delta\tau}{\Delta R^2}\frac{k_f}{k_w}\frac{(2N_r-3)}{(N_r-1)})\phi^n_{w(1,j)} + \frac{Fo_w\Delta\tau}{\Delta Z^2}(\phi^n_{w(N_r,j+1)} + \phi^n_{w(N_r,j-1)}) + \frac{Fo_w\Delta\tau}{\Delta R^2}\frac{k_f}{k_w}(\phi^n_{f(N_r+1,j)} + \frac{N_r}{(N_r-1)}\phi^n_{f(N_r-1,j)}) \quad ...(39)$$

$$\frac{\partial \phi^n_{w(N_{rt},j)}}{\partial R} = \frac{k_p}{k_w}\frac{\partial \phi^n_{p(1,j)}}{\partial R} \qquad n = 0, \tau_{max}, \qquad i = N_{rt} \qquad j = 1, N_z \quad ...(40)$$

$$\frac{\partial^2 \phi_w}{\partial Z^2} + \frac{1}{R}(\frac{\partial}{\partial R}(R\frac{\partial \phi_w}{\partial R})) = \frac{1}{Fo_w}\frac{\partial \phi_w}{\partial \tau} \Rightarrow \frac{\partial^2 \phi_w}{\partial Z^2} + \frac{1}{R}\frac{k_p}{k_w}\frac{\partial \phi_p}{\partial R} + \frac{k_p}{k_w}\frac{\partial^2 \phi_p}{\partial R^2} = \frac{1}{Fo_w}\frac{\partial \phi_w}{\partial \tau}$$

$$\phi^{n+1}_{w(N_{rt},j)} = \begin{Bmatrix} (1-2\frac{Fo_w\Delta\tau}{\Delta Z^2})\phi^n_{w(N_{rt},j)} + \frac{Fo_w\Delta\tau}{\Delta Z^2}(\phi^n_{w(N_{rt},j+1)} + \phi^n_{w(N_{rt},j-1)}) + \frac{Fo_w\Delta\tau}{(N_{rt}-1)\Delta R_p^2}\frac{k_p}{k_w}\phi^n_{p(2,j)} \\ + \frac{Fo_w\Delta\tau}{\Delta R_p^2}\frac{k_p}{k_w}(\phi^n_{p(N_{rt}+1,j)} + \phi^n_{p(N_{rt-1},j)}) - \frac{Fo_w\Delta\tau}{\Delta R_p^2}\frac{(2N_{rt}-1)}{(N_{rt}-1)}\frac{k_p}{k_w}\phi^n_{p(N_{rt},j)} \end{Bmatrix} \quad ...(41)$$

$$\phi^n_{w(N_{rt},j)} = \phi^n_{p(1,j)} \qquad n = 0, \tau_{max}, \qquad i = N_{rt} \qquad j = 1, N_z \quad ...(42)$$

Then, the equation (41), can be written:

$$\phi^{n+1}_{w(N_{rt},j)} = \begin{Bmatrix} (1-2\frac{Fo_w\Delta\tau}{\Delta Z^2} - \frac{Fo_w\Delta\tau}{\Delta R_p^2}\frac{(2N_{rt}-1)}{(N_{rt}-1)}\frac{k_p}{k_w})\phi^n_{w(N_{rt},j)} + \frac{Fo_w\Delta\tau}{\Delta Z^2}(\phi^n_{w(N_{rt},j+1)} + \phi^n_{w(N_{rt},j-1)}) + \\ \frac{Fo_w\Delta\tau}{\Delta R_p^2}\frac{(2N_{rt}-1)}{(N_{rt}-1)}\frac{k_p}{k_w}\phi^n_{p(N_{rt}+1,j)} + \frac{Fo_w\Delta\tau}{\Delta R_p^2}\frac{k_p}{k_w}\phi^n_{p(N_{rt-1},j)} \end{Bmatrix} \quad ...(43)$$

<u>For the PCM</u>

$$\phi^n_{p(i,j)} = \begin{Bmatrix} 0 & \text{Uncharged Process (Cold fluid)} \\ 1 & \text{Charged Process (Hot fluid)} \end{Bmatrix} \quad \text{where } n = 0, \qquad i = 1, N_{rp}, \qquad j = 1, N_z \quad ...(44)$$

$$\frac{\partial \phi^n_{p(i,j)}}{\partial Z} = 0 \qquad n = 0, \tau_{max}, \qquad i = 2, N_{rp}-1, \qquad j = 1 \quad ...(45)$$

$$\frac{2\phi^n_{p(i,2)} - 2\phi^n_{p(i,1)}}{\Delta Z^2} + \frac{\phi^n_{p(i+1,1)} - 2\phi^n_{p(i,1)} + \phi^n_{p(i-1,1)}}{\Delta R_p^2} + \frac{1}{(i-1)\Delta R_p}\frac{\phi^n_{p(i+1,1)} - \phi^n_{p(i,1)}}{\Delta R_p} = \frac{1}{Fo_p}\frac{\phi^{n+1}_{p(i,1)} - \phi^n_{p(i,1)}}{\Delta \tau}$$

$$\phi^{n+1}_{p(i,1)} = (1-2\frac{\Delta\tau Fo_p}{\Delta Z^2} - 2\frac{\Delta\tau Fo_p}{\Delta R_p^2} - \frac{\Delta\tau Fo_p}{(i-1)\Delta R_p^2})\phi^n_{p(i,1)} + 2\frac{\Delta\tau Fo_p}{\Delta Z^2}\phi^n_{p(i,2)} + \frac{\Delta\tau Fo_p}{\Delta R_p^2}\frac{i}{(i-1)}\phi^n_{p(i+1,1)} + \frac{\Delta\tau Fo_p}{\Delta R_p^2}\phi^n_{p(i-1,1)} \quad ...(46)$$

$$\frac{\partial \phi^n_{p(i,j)}}{\partial Z} = 0 \qquad n = 0, \tau_{max}, \qquad i = 2, N_{rp}-1, \qquad j = N_z \quad ...(47)$$

$$\phi^{n+1}_{p(i,N_z)} = (1-2\frac{\Delta\tau Fo_p}{\Delta Z^2} - 2\frac{\Delta\tau Fo_p}{\Delta R_p^2} - \frac{\Delta\tau Fo_p}{(i-1)\Delta R_p^2})\phi^n_{p(i,N_z)} + 2\frac{\Delta\tau Fo_p}{\Delta Z^2}\phi^n_{p(i,N_z-1)} + \frac{\Delta\tau Fo_p}{\Delta R_p^2}\frac{i}{(i-1)}\phi^n_{p(i+1,N_z)} + \frac{\Delta\tau Fo_p}{\Delta R_p^2}\phi^n_{p(i-1,N_z)} \quad ...(48)$$

$$\frac{\partial \phi^n_{p(i,j)}}{\partial R} = 0 \qquad n = 0, \tau_{max}, \qquad i = N_{rp}, \qquad j = 1, N_z \quad ...(49)$$

$$\frac{\partial^2 \phi_p}{\partial Z^2} - \frac{\tan(\psi)}{R}\frac{\partial \phi_p}{\partial Z} + \frac{\partial^2 \phi_p}{\partial R^2} + \frac{1}{R}\frac{\partial \phi_p}{\partial R} = \frac{1}{Fo_p}\frac{\partial \phi_p}{\partial \tau} \Rightarrow \frac{\partial^2 \phi_p}{\partial Z^2} - \frac{\tan(\psi)}{R}\frac{\partial \phi_p}{\partial Z} + \frac{\partial^2 \phi_p}{\partial R^2} = \frac{1}{Fo_p}\frac{\partial \phi_p}{\partial \tau}$$

$$\phi^{n+1}_{p(N_{rp},j)} = \begin{cases} (1-2\frac{\Delta\tau Fo_p}{\Delta Z^2} - 2\frac{\Delta\tau Fo_p}{\Delta R_p^2} - \frac{\Delta\tau Fo_p \tan(\psi)}{(N_{rp}-1)\Delta R_p \Delta Z})\phi^n_{p(N_{rp},j)} + \frac{\Delta\tau Fo_p}{\Delta Z^2}\phi^n_{p(N_{rp},j+1)} \\ +(\frac{\Delta\tau Fo_p \tan(\psi)}{(N_{rp}-1)\Delta R_p \Delta Z} + \frac{\Delta\tau Fo_p}{\Delta Z^2})\phi^n_{p(N_{rp},j-1)} + 2\frac{\Delta\tau Fo_p}{\Delta R_p^2}\phi^n_{p(N_{rp}-1,j)} \end{cases} \quad \ldots (50)$$

$$\frac{\partial \phi^n_{p(1,j)}}{\partial R} = \frac{k_w}{k_p}\frac{\partial \phi^n_{w(N_{rt},j)}}{\partial R} \qquad n = 0, \tau_{max}, \qquad i = N_{rt} \qquad j = 1, N_z \quad \ldots (51)$$

$$\frac{\partial^2 \phi_p}{\partial Z^2} - \frac{\tan(\psi)}{R}\frac{\partial \phi_p}{\partial Z} + \frac{1}{R}\frac{k_w}{k_p}\frac{\partial^2 \phi_w}{\partial R^2} + \frac{k_w}{k_p}\frac{\partial \phi_w}{\partial R} = \frac{1}{Fo_p}\frac{\partial \phi_p}{\partial \tau}$$

$$\phi^{n+1}_{p(1,j)} = \begin{cases} (1-2\frac{\Delta\tau Fo_p}{\Delta Z^2} - \frac{\tan(\psi)}{(i-1)\Delta R_p}\frac{\Delta\tau Fo_p}{\Delta Z})\phi^n_{p(1,j)} + \frac{\Delta\tau Fo_p}{\Delta Z^2}(\phi^n_{p(1,j+1)} + \phi^n_{p(1,j-1)}) + \frac{\tan(\psi)}{(i-1)\Delta R_p}\frac{\Delta\tau Fo_p}{\Delta Z}\phi^n_{p(1,j-1)}) \\ +(\frac{\Delta\tau Fo_p}{(N_{rt}-1)\Delta R_w^2} + \frac{\Delta\tau Fo_p}{\Delta R_w^2})\frac{k_w}{k_p}\phi^n_{w(N_{rt}+1,j)} - \frac{(2N_{rt}-1)}{(N_{rt}-1)}\frac{\Delta\tau Fo_p}{\Delta R_w^2}\frac{k_w}{k_p}\phi^n_{w(N_{rt},j)} + \frac{k_w}{k_p}\frac{\Delta\tau Fo_p}{\Delta R_w^2}\phi^n_{w(N_{rt}-1,j)} \end{cases} \quad \ldots (52)$$

$$\phi^n_{w(N_{rt},j)} = \phi^n_{p(1,j)} \qquad n = 0, \tau_{max}, \qquad i = N_{rt} \qquad j = 1, N_z \quad \ldots (53)$$

Then, the equation (52), can be written:

$$\phi^{n+1}_{p(1,j)} = \begin{cases} (1-2\frac{\Delta\tau Fo_p}{\Delta Z^2} - \frac{\tan(\psi)}{(i-1)\Delta R_p}\frac{\Delta\tau Fo_p}{\Delta Z} - \frac{(2N_{rt}-1)}{(N_{rt}-1)}\frac{\Delta\tau Fo_p}{\Delta R_w^2}\frac{k_w}{k_p})\phi^n_{p(N_{rt},j)} + \frac{\Delta\tau Fo_p}{\Delta Z^2}(\phi^n_{p(N_{rt},j+1)} + \phi^n_{p(N_{rt},j-1)}) \\ + \frac{\tan(\psi)}{(i-1)\Delta R_p}\frac{\Delta\tau Fo_p}{\Delta Z}\phi^n_{p(N_{rt},j-1)} + (\frac{\Delta\tau Fo_p}{(N_{rt}-1)\Delta R_w^2} + \frac{\Delta\tau Fo_p}{\Delta R_w^2})\frac{k_w}{k_p}\phi^n_{w(N_{rt}+1,j)} + \frac{k_w}{k_p}\frac{\Delta\tau Fo_p}{\Delta R_w^2}\phi^n_{w(N_{rt}-1,j)} \end{cases} \quad \ldots (54)$$

3.3. Energy and exergy analysis

The cumulative energy stored in the PCM at any time is calculated as following (Jegadheeswaran and Pohekar, [2010]):

$$E_p = \frac{\pi}{2}(R_o^2 - R_i^2)L\rho_p H_p \quad \ldots(61)$$

Subsequently, the cumulative exergy stored in the PCM at any time can be calculated as follows (Jegadheeswaran and Pohekar, [2010]):

$$Ex_{pm} = E_p\,(1 - \frac{T_\infty}{T_m}) \quad \ldots(62)$$

The exergy transferred by the hot fluid can be evaluated as following (Jegadheeswaran and Pohekar, [2010]),

$$Ex_{fh} = m_{fh}C_{fh}[\,(T_{fhi} - T_{fho}) - T_\infty \,Ln(\frac{T_{fhi}}{T_{fho}})]\,\nabla t \quad \ldots(63)$$

For the cold fluid can be written as,

$$Ex_{fc} = m_{fc}C_{fc}[(T_{fco} - T_{fci}) - T_{\infty} Ln(\frac{T_{fco}}{T_{fci}})] \nabla t \qquad ...(64)$$

3.4 The Exergy Effectiveness (ε)

The exergy efficiency for the hot and cold fluids can be expressed as, (Jegadheeswaran and Pohekar, [2010])

$$\varepsilon_h = \frac{Ex_{fh}}{Ex_{pm}} \quad \text{and} \quad \varepsilon_c = \frac{Ex_{fc}}{Ex_{pm}} \qquad ...(65)$$

4. CASE STUDY DATA

The thermo-physical properties of working fluid, PCM, tubes and the environment as mentioned in tables (1) and (2), (Jegadheeswaran and Pohekar, [2010]):

Table 1. Hot, cold working fluid and paraffin Properties, (Jegadheeswaran and Pohekar, [2010]

Properties	Hot Fluid	Cold Fluid	Properties	Paraffin
Inlet temperature (K)	340	300	Fusion temperature (K)	300.7
Velocity (m/s)	0.01	0.01	Latent heat of fusion (kJ/kg)	206
Thermal conductivity (W/m.K)	0.6605	0.6103	Thermal conductivity (W/m.K)	0.18(solid)/0.19(liquid)
Specific heat (J/kg.K)	4189	4181	Specific heat (kJ/kg.K)	1.8 (solid)/2.4(liquid)
Density (kg/m^3)	979.5	996.5	Density (kg/m^3)	789(solid)/750(liquid)
Thermal diffusion (m^2/s)	1.61*10^{-7}	1.465*10^{-7}	Thermal diffusion (m^2/s)	1.3*10^{-7}
Kinematics viscosity (m^2/s)	4.308*10^{-7}	8.568*10^{-7}	Particle volume fraction, e	0.1

Table 2. Design and operating parameters used in the analysis, (Jegadheeswaran and Pohekar, [2010]

Inner radius of the tube, ri (m)	0. 35	Thermal conductivity (W/m.K)	400
Outer radius of the tube, ro (m)	0.45	Thermal diffusion (m^2/s)	11.57*10^{-5}
Length of the unit, L (m)	1.5	Density (kg/m^3)	8954
Radius of the shell, Ro (m)	0.75	Specific heat (kJ/kg.K)	0.384
Environment temperature, T_{∞} (K)	298		

5. VALIDATION OF THE PRESENT MODEL

A Q.Basic computer program is developed to solve the present problem. Figure (4), shows a comparison of the obtained dimensionless temperature for the fluid (outlet fluid temperature to the environment temperature) that obtained by the present analysis and that computed by Keshavarz et al, [2003], for the data in the table (4). The average relative difference between the two values over the whole dataset is less than 2%. So, it can be concluded that the present analysis are correctly achieved and formulated.

Table 3. Thermophysical properties of Figure (2), Keshavarz et al, [2003]

Properties	PCM (Lithium fluoride)	Fluid (Liquid sodium)
Latent heat of fusion (kJ/kg)	550	-
Density (kg/m^3)	2300	929.1
Specific heat (kJ/kg.K)	2.87 (solid)/ 1.99(liquid)	1.38
Thermal conductivity (W/m.K)	3.5	86.2
Melting Temperature / Environment Temperature	1.5	-
Environment Temperature (K)	298	

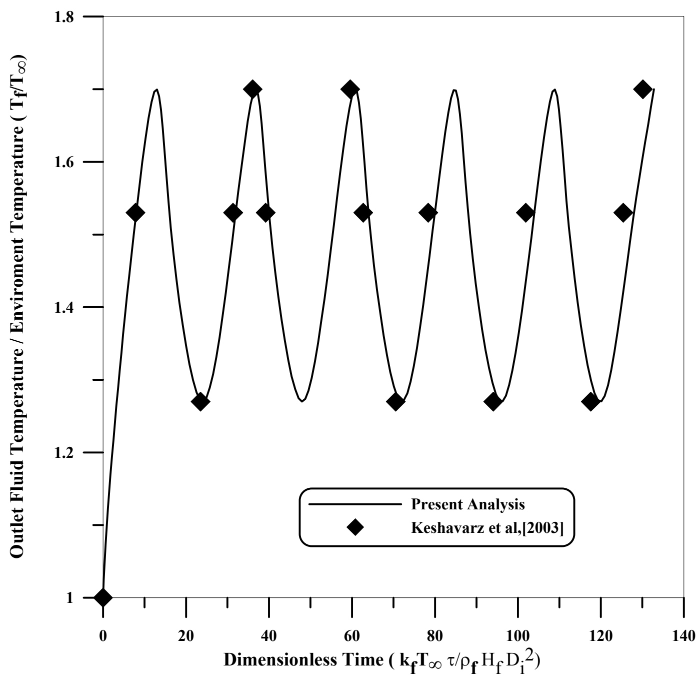


Figure (2), History of the outlet fluid temperature obtained by the present analysis and the dimensionless outlet fluid computed by the analysis by Keshavarz et al, 2003.

6. RESULTS AND DISCUSSION

Figures (3) and (4) show the variation of the hot fluid exergy effectiveness and the heat transferred from the hot fluid to the PCM per unit volume of PCM respectively against the time for different values of (ψ). From these figures, the effectiveness and the heat transfer per unit volume increased with decreasing the value of (ψ), for positive value of the (ψ) (clockwise direction), the increase of (ψ) will non largely effected on the effectiveness and the heat transfer per unit volume of PCM compared with the negative value, because at positive value of (ψ), the quality of PCM (volume and heat capacity of PCM) near the entrance lower than the existing. Therefore, more heat is transfer at entrance (small volume of PCM), compared with low value of heat transfer at existence (large volume of PCM). Same behaviour can be noted for the cold fluid for parallel flow in figures (5) and (6) for both cold fluid effectiveness and the heat transferred to the cold fluid from PCM per unit volume of PCM. The exergy effectiveness and the heat transfer per unit volume of PCM for the cold fluid increased with decreased values of (ψ). Also, increasing (ψ) (from negative to positive value) will decrease the value of the effectiveness and the heat transfer value per unit volume of PCM. Figures (7) and (8) show the effectiveness and the heat transfer for the counter flow case are higher than that for the parallel case due to larger amount of heat is transferred for the same properties and for the same time duration. The effect of the fluid velocity of the cold and hot fluid on the effectiveness are plotted in the Figures (9) and (10) for hot and cold fluid respectively for three values of (ψ) (positive, negative and zero values). The difference between the values of effectiveness for different values of (ψ) decrease with increase the fluid velocity due to reducing the time for the heat transfer between fluids and PCM, then the heat transfer become less with decreasing time (increasing of the fluid velocity). For the high velocity, the effect of the (ψ) is low and this effect will decrease with increase the velocity of fluid. Figure (11) shows the effect of the (ψ) on the effectiveness of the hot fluid and cold fluid (parallel and counter flow). From this figure, it can be seen that the large effectiveness value is obtained for the smaller negative value of (ψ = - 0.13 deg), under constant velocity. The difference between the effectiveness of the cold fluid in parallel form and counter flow form decreased with increased the value of (ψ) in positive range.

7. CONCLUSIONS:

In this paper, numerical analysis by utilizing finite difference method is adopted to study the two dimensional transient heat transfers through thermal energy storage. From the results it is concluded that:

1- Decreasing the value of cone angle (ψ) leads to increase the effectiveness of the hot and cold fluids in TES.

2- The difference between the effectiveness (hot and parallel cold and counter cold) increased with decreased value of (ψ) in the negative range.

3- The effect of (ψ) on the effectiveness and the heat transfer decreased with increased value of (ψ) in the positive range.

4- For the optimum selection of the storage thermal system, the values of (ψ) preferred negative, and the cold fluid is counter flow.

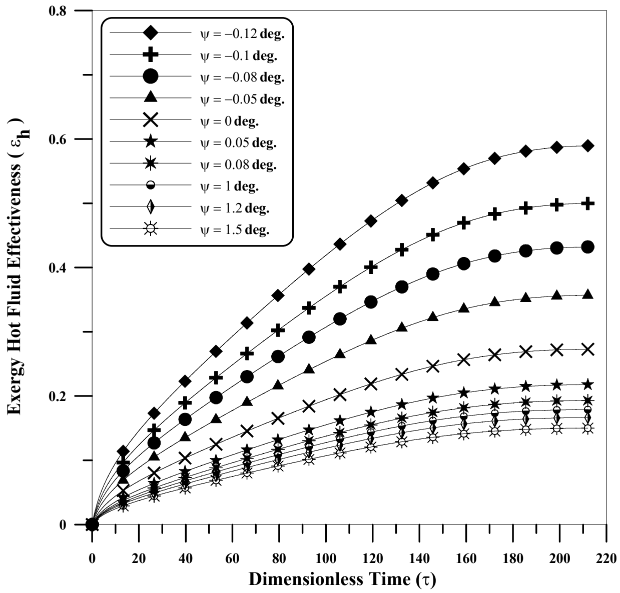


Figure (3), History of hot fluid effectiveness for different values of (ψ)

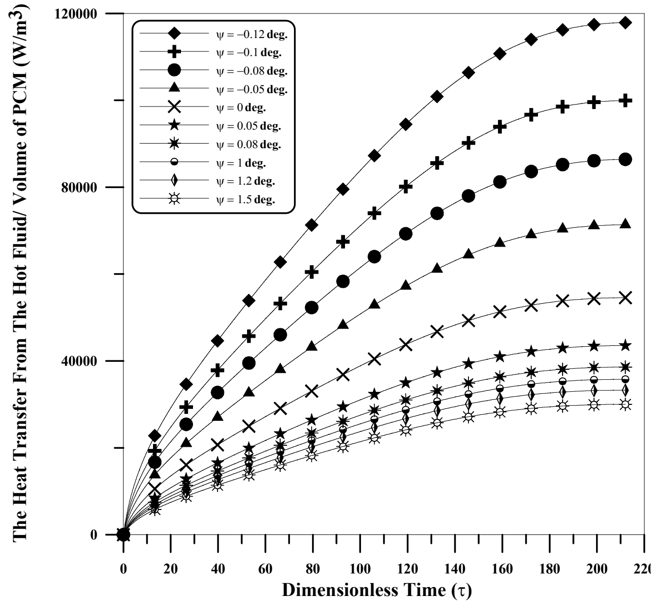


Figure (4), History of hot fluid heat transfer per unit volume of PCM for different values of (ψ)

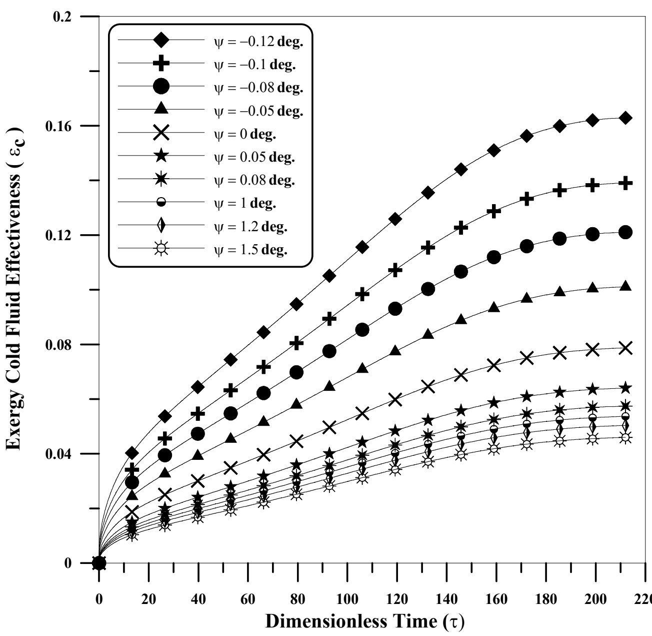


Figure (5), History of cold fluid effectiveness (parallel flow) for different values of (ψ)

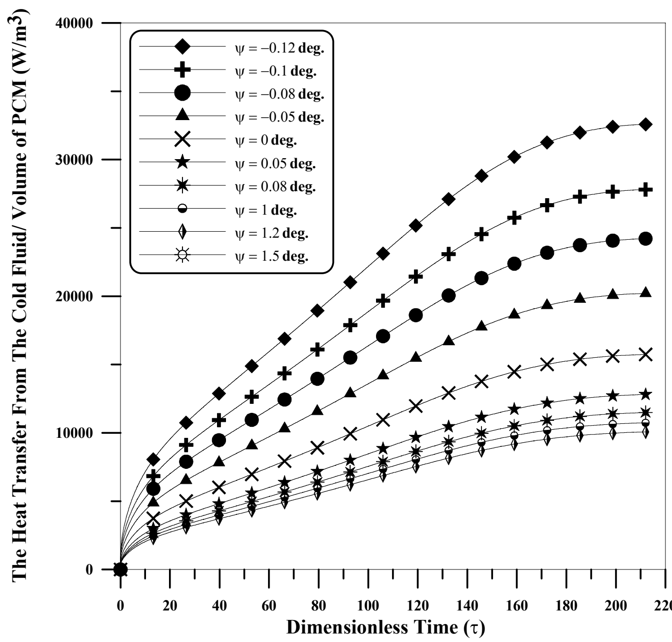


Figure (6), History of cold fluid heat transfer (parallel flow) per sunit volume of PCM for different values of (ψ)

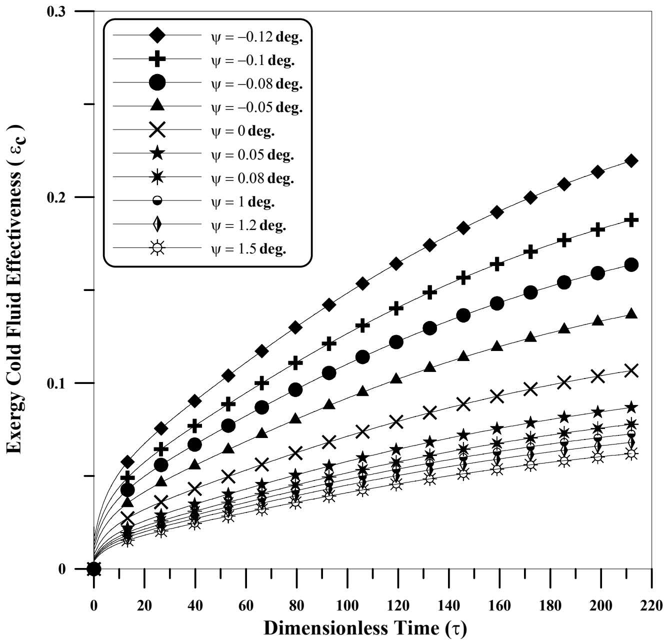


Figure (7), History of cold fluid effectiveness (counter flow) for different values of (ψ)

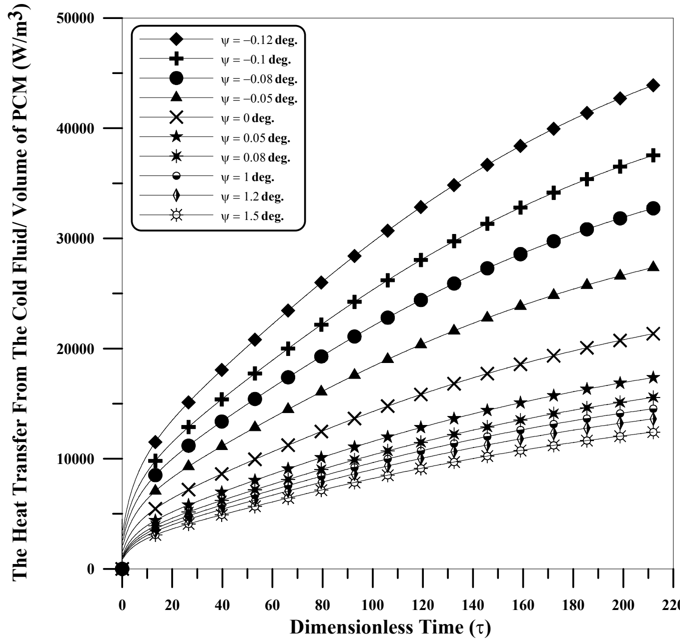


Figure (8), History of cold fluid heat transfer (counter flow) per unit volume of PCM for different values of (ψ)

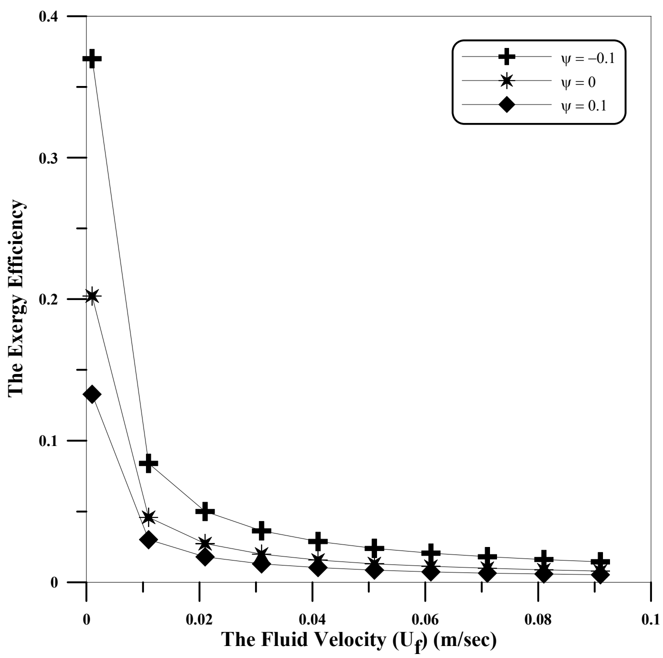


Figure (9), History of hot fluid exergy effectiveness for different values of (ψ)

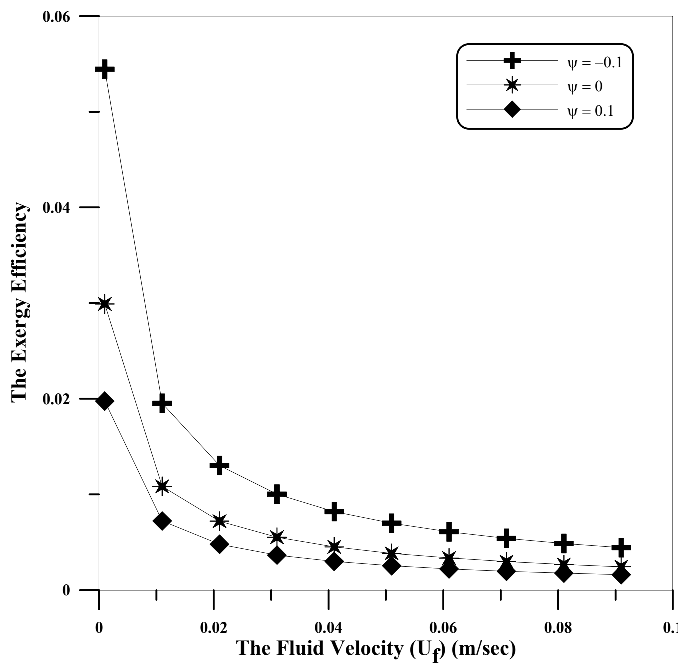


Figure (10), History of cold fluid exergy effectiveness for different values of (ψ)

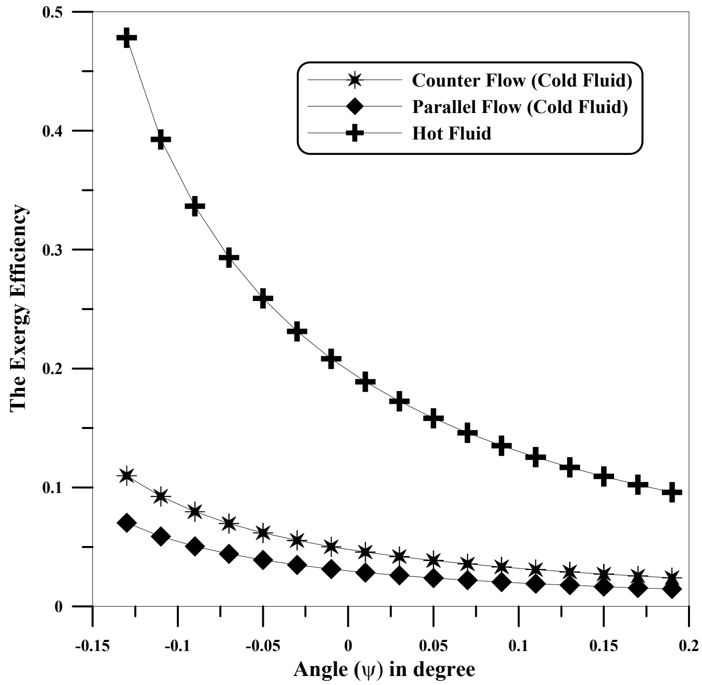


Figure (11), History of fluid exergy effectiveness (hot and cold) for different values of (ψ)

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APPENDIX – A

1. The governing equation for the fluid

The main governing equation for the fluid is derived from Navier- Stoke energy equation in two dimensional cylindrical coordinates (Figure (1)), [Deyi Shang, 2006], as following:

$$\frac{D(\rho_f C_f T_f)}{Dt} = \Phi + \nabla.(k_f \nabla T_f) \pm Q''' \qquad \text{…(A1)}$$

Where $\Phi = \mu(2(\xi_{rr}^2 + \xi_{zz}^2) + \xi_{rz}^2)$,and $\xi_{rr} = \frac{\partial u_r}{\partial r},\ \xi_{zz} = \frac{\partial u_z}{\partial z},\ \xi_{rz} = \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}$ …(A2)

The fluid flow assumed without heat generation (Q'''=0) as following:

$$\rho_f C_f \left[\frac{\partial T_f}{\partial t} + (u.\nabla)T_f\right] = k_f \nabla^2 T_f + \mu(2(\xi_{rr}^2 + \xi_{zz}^2) + \xi_{rz}^2) \qquad \text{...(A3)}$$

$$\rho_f C_f \left[\frac{\partial T_f}{\partial t} + u_r \frac{\partial T_f}{\partial r} + u_z \frac{\partial T_f}{\partial z}\right] = k_f \left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_f}{\partial r}\right) + \frac{\partial^2 T_f}{\partial z^2}\right) + \mu\left(2\left(\left(\frac{\partial u_r}{\partial r}\right)^2 + \left(\frac{\partial u_z}{\partial z}\right)^2\right) + \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)^2\right) \qquad \text{...(A4)}$$

If the fluid velocity in the z- direction very large from fluid velocity in r- direction,

$u_f = u_r + u_z$ where $u_z >> u_r$ then $u_f \approx u_z$ …(A5)

The fluid flow approximately with constant velocity (u_f),

$$u_r \frac{\partial T_f}{\partial r} >> u_z \frac{\partial T_f}{\partial z} \quad \text{and} \quad \frac{\partial u_r}{\partial r} \approx 0,\ \frac{\partial u_z}{\partial z} \approx 0,\ \frac{\partial u_r}{\partial z} \approx 0,\ \frac{\partial u_z}{\partial r} \approx 0 \qquad \text{...(A6)}$$

$$\frac{\partial T_f}{\partial t} + u_z \frac{\partial T_f}{\partial z} = \frac{k_f}{\rho_f C_f}\left(\frac{\partial^2 T_f}{\partial r^2} + \frac{1}{r}\frac{\partial T_f}{\partial r} + \frac{\partial^2 T_f}{\partial z^2}\right) \qquad \text{...(A7)}$$

$$\frac{\partial T_f}{\partial t} + u_f \frac{\partial T_f}{\partial z} = \alpha_f\left(\frac{\partial^2 T_f}{\partial r^2} + \frac{1}{r}\frac{\partial T_f}{\partial r} + \frac{\partial^2 T_f}{\partial z^2}\right) \qquad \text{...(A8)}$$

$$\frac{\partial^2 \phi_f}{\partial Z^2} + \frac{\partial^2 \phi_f}{\partial R^2} + \frac{1}{R}\frac{\partial \phi_f}{\partial R} = \frac{t_{con} r_i^2}{\alpha_f}\frac{\partial \phi_f}{\partial \tau} + \frac{u_f\, r_i}{\alpha_f}\frac{\partial \phi_f}{\partial Z} \qquad \text{...(A9)}$$

$$\frac{\partial^2 \phi_f}{\partial Z^2} + \frac{\partial^2 \phi_f}{\partial R^2} + \frac{1}{R}\frac{\partial \phi_f}{\partial R} = \frac{1}{Fo_f}\frac{\partial \phi_f}{\partial \tau} + Re_f\, Pr_f \frac{\partial \phi_f}{\partial Z} \qquad \text{...(A10)}$$

2. The energy equation for the cylinder wall

Applying the energy balance around the control volume inside the cylinder wall, as following:

$$\begin{bmatrix}\text{Change of internal energy}\\ \text{through clyinder wall with time}\end{bmatrix}=\begin{bmatrix}\text{heat transfer through}\\ \text{cylinder wall in z-direction}\end{bmatrix}+\begin{bmatrix}\text{heat transfer through}\\ \text{cylinder wall in z-direction}\end{bmatrix} \quad ...(A11)$$

$$\frac{\partial}{\partial z}(k_w A_{wcond}\frac{\partial T_w}{\partial z})\Delta z+\frac{\partial}{\partial r}(k_w A_{wconv}\frac{\partial T_w}{\partial r})\Delta r=\frac{\partial}{\partial t}(\rho_w C_w A_{wcond}\Delta z T_w) \quad ...(A12)$$

$$\frac{\partial}{\partial z}(k_w 2\pi r\Delta r\frac{\partial T_w}{\partial z})\Delta z+\frac{\partial}{\partial r}(k_w 2\pi r\Delta z\frac{\partial T_w}{\partial r})\Delta r=\frac{\partial}{\partial t}(\rho_w C_w 2\pi r\Delta r\Delta z T_w) \quad ...(A13)$$

$$\frac{\partial}{\partial z}(k_w\frac{\partial T_w}{\partial z})+\frac{1}{r}\frac{\partial}{\partial r}(k_w r\frac{\partial T_w}{\partial r})=\frac{\partial}{\partial t}(\rho_w C_w T_w) \quad ...(A14)$$

$$\frac{\partial^2 T_w}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T_w}{\partial r})=\frac{1}{\alpha_w}\frac{\partial T_w}{\partial t} \quad \Rightarrow \quad \frac{\partial^2 T_w}{\partial z^2}+\frac{\partial^2 T_w}{\partial r^2}+\frac{1}{r}\frac{\partial T_w}{\partial r}=\frac{1}{\alpha_w}\frac{\partial T_w}{\partial t} \quad ...(A15)$$

$$\frac{\partial^2 T_w}{\partial z^2}+\frac{\partial^2 T_w}{\partial r^2}+\frac{1}{r}\frac{\partial T_w}{\partial r}=\frac{1}{\alpha_w}\frac{\partial T_w}{\partial t} \quad ...(A16)$$

$$\frac{\partial^2 \phi_w}{\partial Z^2}+\frac{\partial^2 \phi_w}{\partial R^2}+\frac{1}{R}\frac{\partial \phi_w}{\partial R}=\frac{r_i^2}{t_{con}\alpha_w}\frac{\partial \phi_w}{\partial \tau} \quad ...(A17)$$

$$\frac{\partial^2 \phi_w}{\partial Z^2}+\frac{\partial^2 \phi_w}{\partial R^2}+\frac{1}{R}\frac{\partial \phi_w}{\partial R}=\frac{1}{Fo_w}\frac{\partial \phi_w}{\partial \tau} \quad ...(A18)$$

3. The energy equation for PCM

Applying the energy balance around the control volume inside the PCM material, as following:

$$\begin{bmatrix}\text{Change of internal energy}\\ \text{through PCM with time}\end{bmatrix}=\begin{bmatrix}\text{heat transfer through}\\ \text{PCM in z-direction}\end{bmatrix}+\begin{bmatrix}\text{heat transfer through}\\ \text{PCM in z-direction}\end{bmatrix} \quad ...(A19)$$

$$\frac{\partial}{\partial z}(k_p A_{pcond}\frac{\partial T_p}{\partial z})\Delta z+\frac{\partial}{\partial r}(k_p A_{pconv}\frac{\partial T_p}{\partial r})\Delta r=\frac{\partial}{\partial t}(\rho_p C_p A_{pcond}\Delta z T_p) \quad ...(A20)$$

$$A_{pcond}=2\pi r\Delta r, \quad A_{pconv}=2\pi r\Delta z, \quad V_p=2\pi r\Delta r\Delta z \quad \text{and} \quad r=R_i+(L-z)\tan(\psi) \quad ...(A21)$$

$$\frac{\partial^2 \phi_p}{\partial Z^2}-\frac{\tan(\psi)}{R}\frac{\partial \phi_p}{\partial Z}+\frac{\partial^2 \phi_p}{\partial R^2}+\frac{1}{R}\frac{\partial \phi_p}{\partial R}=\frac{1}{Fo_p}\frac{\partial \phi_p}{\partial \tau} \quad ...(A22)$$

APPENDIX – B

The finite difference analysis for the governing equation:

For the fluid

$$\frac{\phi^n_{f(i,j+1)}-2\phi^n_{f(i,j)}+\phi^n_{f(i,j-1)}}{\Delta Z^2}+\frac{\phi^n_{f(i+1,j)}-2\phi^n_{f(i,j)}+\phi^n_{f(i-1,j)}}{\Delta R^2}+\frac{1}{(i-1)\Delta R}\frac{\phi^n_{f(i+1,j)}-\phi^n_{f(i,j)}}{\Delta R}=\frac{1}{Fo_f}\frac{\phi^{n+1}_{f(i,j)}-\phi^n_{f(i,j)}}{\Delta \tau}+Pe_f\frac{\phi^n_{f(i,j)}-\phi^n_{f(i,j-1)}}{\Delta Z}$$

$$\left\{\begin{array}{l}\phi^{n+1}_{f(i,j)} = \phi^{n}_{f(i,j)} - \dfrac{Fo_f \Delta\tau Pe_f}{\Delta Z}(\phi^{n}_{f(i,j)} - \phi^{n}_{f(i,j-1)}) + \dfrac{Fo_f \Delta\tau}{(i-1)\Delta R^2}(\phi^{n}_{f(i+1,j)} - \phi^{n}_{f(i,j)}) \\ \dfrac{Fo_f \Delta\tau}{\Delta Z^2}(\phi^{n}_{f(i,j+1)} - 2\phi^{n}_{f(i,j)} + \phi^{n}_{f(i,j-1)}) + \dfrac{Fo_f \Delta\tau}{\Delta R^2}(\phi^{n}_{f(i+1,j)} - 2\phi^{n}_{f(i,j)} + \phi^{n}_{f(i-1,j)})\end{array}\right\}$$

$$\phi^{n+1}_{f(i,j)} = \left\{\begin{array}{l}(1 - \dfrac{Fo_f \Delta\tau Pe_f}{\Delta Z} - \dfrac{Fo_f \Delta\tau}{(i-1)\Delta R^2} - 2\dfrac{Fo_f \Delta\tau}{\Delta Z^2} - 2\dfrac{Fo_f \Delta\tau}{\Delta R^2})\phi^{n}_{f(i,j)} + (\dfrac{Fo_f \Delta\tau Pe_f}{\Delta Z} + \dfrac{Fo_f \Delta\tau}{\Delta Z^2})\phi^{n}_{f(i,j-1)} \\ + \dfrac{Fo_f \Delta\tau}{\Delta R^2}\dfrac{i}{(i-1)}\phi^{n}_{f(i+1,j)} + \dfrac{Fo_f \Delta\tau}{\Delta Z^2}\phi^{n}_{f(i,j+1)} + \dfrac{Fo_f \Delta\tau}{\Delta R^2}\phi^{n}_{f(i-1,j)}\end{array}\right\} \quad \text{...(1B)}$$

$$(1 - \frac{Fo_f \Delta\tau Pe_f}{\Delta Z} - \frac{Fo_f \Delta\tau}{(i-1)\Delta R^2} - 2\frac{Fo_f \Delta\tau}{\Delta Z^2} - 2\frac{Fo_f \Delta\tau}{\Delta R^2}) \geq 0$$

$$\Delta\tau \leq \frac{1}{Fo_f(\dfrac{Pe_f}{\Delta Z} + \dfrac{1}{(i-1)\Delta R^2} + \dfrac{2}{\Delta Z^2} + \dfrac{2}{\Delta R^2})} \quad \text{...(2B)}$$

For the cylinder wall

$$\frac{\phi^{n}_{w(i,j+1)} - 2\phi^{n}_{w(i,j)} + \phi^{n}_{w(i,j-1)}}{\Delta Z^2} + \frac{\phi^{n}_{w(i+1,j)} - 2\phi^{n}_{w(i,j)} + \phi^{n}_{w(i-1,j)}}{\Delta R_w^2} + \frac{1}{(i-1)\Delta R_w}\frac{\phi^{n}_{w(i+1,j)} - \phi^{n}_{w(i,j)}}{\Delta R_w} = \frac{1}{Fo_w}\frac{\phi^{n+1}_{w(i,j)} - \phi^{n}_{w(i,j)}}{\Delta\tau}$$

$$\phi^{n+1}_{w(i,j)} = (1 - 2\frac{\Delta\tau Fo_w}{\Delta Z^2} - 2\frac{\Delta\tau Fo_w}{\Delta R_w^2} - \frac{\Delta\tau Fo_w}{(i-1)\Delta R_w^2})\phi^{n}_{w(i,j)} + \frac{\Delta\tau Fo_w}{\Delta Z^2}(\phi^{n}_{w(i,j+1)} + \phi^{n}_{w(i,j-1)}) + \frac{\Delta\tau Fo_w}{\Delta R_w^2}[\frac{i}{(i-1)}\phi^{n}_{w(i+1,j)} + \phi^{n}_{w(i-1,j)}] \quad \text{...(3B)}$$

$$\Delta\tau \leq \frac{1}{Fo_w(\dfrac{2}{\Delta Z^2} + \dfrac{2}{\Delta R_w^2} + \dfrac{1}{(i-1)\Delta R_w^2})} \quad \text{...(4B)}$$

For the PCM

$$\frac{\phi^{n}_{p(i,j+1)} - 2\phi^{n}_{p(i,j)} + \phi^{n}_{p(i,j-1)}}{\Delta Z^2} - \frac{\tan(\psi)}{(i-1)\Delta R_p}\frac{\phi^{n}_{p(i,j)} - \phi^{n}_{p(i,j-1)}}{\Delta Z} + \frac{\phi^{n}_{p(i+1,j)} - 2\phi^{n}_{p(i,j)} + \phi^{n}_{p(i-1,j)}}{\Delta R_p^2} + \frac{1}{(i-1)\Delta R_p}\frac{\phi^{n}_{p(i+1,j)} - \phi^{n}_{p(i,j)}}{\Delta R_p} = \frac{1}{Fo_p}\frac{\phi^{n+1}_{p(i,j)} - \phi^{n}_{p(i,j)}}{\Delta\tau}$$

$$\phi^{n+1}_{p(i,j)} = \left\{\begin{array}{l}(1 - 2\dfrac{\Delta\tau Fo_p}{\Delta Z^2} - 2\dfrac{\Delta\tau Fo_p}{\Delta R_p^2} - \dfrac{\Delta\tau Fo_p \tan(\psi)}{(i-1)\Delta R_p \Delta Z} - \dfrac{\Delta\tau Fo_p}{(i-1)\Delta R_p^2})\phi^{n}_{p(i,j)} + \dfrac{\Delta\tau Fo_p}{\Delta Z^2}\phi^{n}_{p(i,j+1)} \\ + (\dfrac{\Delta\tau Fo_p \tan(\psi)}{(i-1)\Delta R_p \Delta Z} + \dfrac{\Delta\tau Fo_p}{\Delta Z^2})\phi^{n}_{p(i,j-1)} + \dfrac{\Delta\tau Fo_p}{\Delta R_p^2}\dfrac{i}{(i-1)}\phi^{n}_{p(i+1,j)} + \dfrac{\Delta\tau Fo_p}{\Delta R_p^2}\phi^{n}_{p(i-1,j)}\end{array}\right\} \quad \text{...(5B)}$$

$$\Delta\tau \leq \frac{1}{Fo_p(\dfrac{2}{\Delta Z^2} + \dfrac{2}{\Delta R_p^2} + \dfrac{\tan(\psi)}{(i-1)\Delta R_p \Delta Z} + \dfrac{1}{(i-1)\Delta R_p^2})} \quad \text{...(6B)}$$